

## CORRESPONDENCE

## Comments on "Fields of Correlation Assembly—A Numerical Analysis Technique"

C. EUGENE BUELL

Kaman Nuclear, Colorado Springs, Colo.

The following comments refer to a recent paper by Danard, Holl, and Clark (1968).

*First:* It would appear that the "structure" function  $\sigma^2_\Delta(R)$  and the "reliability"  $B(R)=1/(2S^2_\Delta(R))$  where  $S^2_\Delta(R)=\sigma^2_w+\sigma^2_\Delta(R)$ ,  $\sigma^2_w$  being the error variance of an observed wind component, are considered as functions of the radial distance,  $R$ , from the point of observation to the point on the grid to which the gradient component information is extrapolated. (See also fig. 3 where these functions are illustrated.) We are dealing with components of the gradient of a scalar function  $\Delta\epsilon=\Delta(Z-Z_g)$ , where  $\Delta$  stands for a difference in either of the grid directions, i.e., the finite difference version of  $\partial\epsilon/\partial x$  or  $\partial\epsilon/\partial y$ . It is only under the most unusual circumstances that the structure function, or the correlation coefficient function, of such a gradient component would be dependent only on radial distance. Such structure functions or correlation coefficient functions are highly dependent on the azimuth of the line joining the points.

We would hesitate to guess the exact form of the correlation coefficient functions for  $\Delta(Z-Z_g)\sim\partial(Z-Z_g)/\partial x$  or  $\sim\partial(Z-Z_g)/\partial y$ , but they must bear a close resemblance to the correlation coefficient functions for wind components. These correlation coefficient functions are roughly like

$$r_{uu'}=(1-y^2/L^2)\exp(-R^2/2L^2)$$

$$r_{vv'}=(1-x^2/L^2)\exp(-R^2/2L^2)$$

$$R^2=x^2+y^2$$

where  $L$  is an appropriate scale parameter, and  $(x,y)$  are the coordinate differences of the point  $P$  where  $(u,v)$  are measured and the point  $P'$  where  $(u',v')$  are measured. We have verified this character of the correlation coefficient function for wind components from pole to pole, from the ground to well into the stratosphere, and all seasons of the year. It certainly holds up wherever the geostrophic approximation is even reasonably valid (so we must except the equatorial region). (See Buell, 1959, 1960, and 1963.) One must quickly add that the  $(u,v)$  components are exactly those of the  $(x,y)$  coordinates so that if the coordinates are rotated, the correlation coefficient functions above rotate with them. There is nothing sacred about the

fact that  $u$  is pointed toward east and  $v$  toward north. The "invariant" correlation coefficient functions would be for "longitudinal" and "transverse" wind components as determined by the line  $PP'$ .

An aspect of the situation lies in the fact that if the scalar field  $f(x,y)$  is isotropic, its correlation coefficient function is dependent only on  $R$ . Now  $\partial f/\partial x=g(x,y)$  is another scalar field, but if  $f(x,y)$  is isotropic, then  $g(x,y)$  may be very far from an isotropic field. Gandin (1965, p. 16ff.), covers isotropy for scalar fields very well, but for vector fields the situation is somewhat different. If  $f(x,y)$  is an isotropic scalar field, then the fields of  $\partial f/\partial x$ ,  $\partial f/\partial y$  together form an isotropic vector field (approximated in the wind situation) but neither  $\partial f/\partial x$  nor  $\partial f/\partial y$  individually is an isotropic scalar field.

We were amazed to find that Gandin (1965, pp. 49–52) describes the structure function for wind in terms of radial distance between points. Since he compares his results with those of Durst (1954), we presume that Durst's vector correlation coefficient is being used. The situation is not that simple. Nothing short of a correlation or covariance tensor is needed to adequately describe the statistical structure of a vector field. We cannot agree with Gandin's (1965) assertion (pp. 50–51) that wind data (generally) are unreliable. Our data were carefully scanned for possible gross errors. In all cases the structure of the correlation coefficient tensor was remarkably well defined.

*Second:* We would like to suggest that the procedure can be greatly simplified by using a straight statistical estimate to extrapolate the height of the isobaric surface:

$$z'_{est}=az+bu+cv$$

where standardized variables were being used. Since wind components at a point are nearly uncorrelated with height and each other, this will be approximately

$$z'_{est}=r_{zz'}Z+r_{uz'}u+r_{vz'}v.$$

The correlations involved (even in the general case) are well known (Buell, 1958a, 1958b, 1960) so that the appropriate weighting factors for the construction of the final field estimates are easily obtained. The situation is readily adapted to include points from which  $Z$  only,  $(U,V)$ , only, or  $Z$  and  $(U,V)$  are observed. Particularly in the last case, the approximation holds up well at much

larger distances than the tangent plane approximation. At low latitude, especially in summer, the full form of  $a$ ,  $b$ ,  $c$  are needed since  $r_{uz'}(0) = r_{uz}$  is appreciably not zero.

In even the simplest case the coefficients  $r_{uz'}$ ,  $r_{vz'}$  are strongly direction dependent,  $r_{uz'} = r_{uz'}(R, \theta)$ ,  $r_{vz'} = r_{vz'}(R, \theta)$ . The wind provides more information on height in the direction perpendicular to the wind component than in the component direction (in the sense of a standardized variable). In fact, if the subscript  $t$  indicates the component transverse to the direction  $PP'$ , then  $r_{tz'} \simeq r_{tz'}(R)$ , a function of only the distance  $PP'$  (excluding low-latitude summer situations).

#### REFERENCES

- Buell, C. E., "The Correlation between Wind and Height on an Isobaric Surface," *Journal of Meteorology*, Vol. 15, No. 3, June 1958a, pp. 309-316.
- Buell, C. E., "The Correlation between Wind and Height on an Isobaric Surface: II. Summer Data," *Journal of Meteorology*, Vol. 15, No. 6, Dec. 1958b, pp. 502-512.
- Buell, C. E., "An Application of Turbulence Theory to the Synoptic Scale Phenomena of the Atmosphere," *Final Report*, 4 vols., Contract No. DA-04-495-ORD-1195, KN-96-59-16(FR), Kaman Nuclear, Colorado Springs, Sept. 1959.
- Buell, C. E., "The Structure of Two-Point Wind Correlations in the Atmosphere," *Journal of Geophysical Research*, Vol. 65, No. 10, Oct. 1960, pp. 3353-3366.
- Buell, C. E., "Two-Point Variability of Wind," *Final Report* 3 vols., Contract No. AF19(604)7282, KN-173-62-1(FR), Kaman Nuclear, Colorado Springs, July 1963.
- Danard, M. B., Holl, M. M., and Clark, J. R., "Fields by Correlation Assembly—A Numerical Analysis Technique," *Monthly Weather Review*, Vol. 96, No. 3, Mar. 1968, pp. 141-149.
- Durst, C. S., "Variation of Wind With Time and Distance," *Geophysical Memoirs*, Vol. 12, No. 93, Meteorological Office, London, 1954, 32 pp.
- Gandin, L. S., *Objective Analysis of Meteorological Fields*, (*Ob'ektivnyi analiz meteorologicheskikh polei*, 1963), Israel Program for Scientific Translations, Jerusalem, 1965, 242 pp.

#### Reply

M. B. DANARD,<sup>1</sup> M. M. HOLL, AND J. R. CLARK

Meteorology International Inc., Monterey, Calif.

With respect to Dr. Buell's first comment, one of the authors (Danard, 1965) has measured  $S_{\Delta}^2$  directly in the lower troposphere (below 10,000 ft). There was no appreciable difference between the values for the along- and across-wind directions over a distance of 15 mi. This conclusion would not necessarily be true over longer distances and would probably not be true in the upper troposphere. Nevertheless, the assumption that  $S_{\Delta}^2$  is a function of radial distance only is probably justified for the analysis of sea-level pressure. Moreover, it is not likely that this assumption would lead to serious error at 500 mb.

As for the second comment, it should be pointed out that the height or pressure is extrapolated to the nearest grid point only. At 60°N this is a maximum distance of only 270 km. It is even less at lower latitudes. Dr. Buell's suggestion may produce a more accurate estimate. However, it would not likely differ significantly from that produced by our method, because of the relatively small distances involved.

While both Dr. Buell's suggestions have merit, it is not obvious that the improvement would be sufficient to justify the more complicated computer programming required. Since objective analyses are performed routinely on a real-time basis, simplicity of method is desirable. Nevertheless, the authors wish to thank Dr. Buell for bringing up these points for discussion.

#### REFERENCE

- Danard, M. B., "On the Dependence of Wind Variability on Surface Wind Speed, Richardson Number and Height above Terrain," *Journal of Applied Meteorology*, Vol. 4, No. 3, June 1965, pp. 394-399.

<sup>1</sup> Present affiliation: University of Waterloo, Waterloo, Ontario, Canada.

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